Inverse Trigonometric Functions

Question1

$\sec^2\left(\tan^{-1} 2\right)+\operatorname{cosec}^2\left(\cot^{-1} 3\right)=$

KCET 2025

Options:

A. 1

B. 5

C. 15

D. 10

Answer: C

Solution:

$$
\begin{aligned}
& =1+\tan^2\left(\tan^{-1} 2\right)+1+\cot^2\left(\cot^{-1} 3\right) \\
& =1+4+1+9=15
\end{aligned}
$$

Question2

$2 \cos^{-1} x=\sin^{-1}\left(2 x \sqrt{1-x^2}\right)$ **is valid for all values of ' x ' satisfying**

KCET 2025

Options:

A. $0 \leq x \leq \frac{1}{\sqrt{2}}$

B. $-1 \leq x \leq 1$

C. $0 \leq x \leq 1$

D. $\frac{1}{\sqrt{2}} \leq x \leq 1$

Answer: D

Solution:

To determine the values of 'x' that satisfy the equation $2\cos^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, let's analyze the components:

Let $x = \cos\theta$. This implies $\theta = \cos^{-1} x$.

The equation can be expressed as $\sin^{-1}(2\cos\theta\sin\theta)$, which simplifies to $\sin^{-1}(\sin 2\theta)$. Hence, $2\theta = \sin^{-1}(\sin 2\theta)$.

For $\sin^{-1}(\sin 2\theta)$ to equal $2\cos^{-1} x$, i.e., 2θ, 2θ must lie within the range $[0, \pi]$.

Given:

$\theta = \cos^{-1} x$, the valid range for θ is $[0, \pi/4]$.

Therefore, $\cos^{-1} x \in [0, \pi/4]$.

This implies:

x lies within $\left[\frac{1}{\sqrt{2}}, 1\right]$ because $\cos(\pi/4) = \frac{1}{\sqrt{2}}$.

Thus, the equation is satisfied for values of x in the interval $\left[\frac{1}{\sqrt{2}}, 1\right]$.

Question3

Let $f : R \to R$ be given $f(x) = \tan x$. Then, $f^{-1}(1)$ is

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Options:

A. $\frac{\pi}{4}$

B. $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$

C. $\frac{\pi}{3}$

D. $\left\{n\pi + \frac{\pi}{3} : n \in Z\right\}$

Answer: A

Solution:

$\because f(x) = \tan x$

Since, f^{-1} is inverse of f

$$
\begin{array}{ll}
\Rightarrow & f\left(f^{-1}(1)\right) = 1 \\
\Rightarrow & \tan\left(f^{-1}(1)\right) = 1 \\
\text{But} & \tan\left(\frac{\pi}{4}\right) = 1 \\
\Rightarrow & \tan\left(f^{-1}(1)\right) = \tan\left(\frac{\pi}{4}\right) \\
\therefore & f^{-1}(l) = \frac{\pi}{4}
\end{array}
$$

Question4

If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $x(y+z) + y(z+x) + z(x+y)$ equals to

KCET 2024

Options:

A. 0

B. 1

C. 6

D. 2

Answer: C

Solution:

Given, $\cos^{-1}(x)+\cos^{-1}(y)+\cos^{-1}(z)=3\pi$

As, we know that $0 \leq \cos^{-1}(x) \leq \pi$

Thus, the maximum value of $\cos^{-1}(x)=\pi$

satisfies the given equation.

$$\begin{aligned} & x=y=z=\cos \pi=-1 \\ \text{Now, } & x(y+z)+y(z+x)+z(x+y) \\ & =xy+zx+yz+xy+zx+yz \\ & =2(xy+yz+zx) \\ & =2(1+1+1)=6 \end{aligned}$$

Question5

If $2 \sin^{-1} x-3 \cos^{-1} x=4, x \in[-1,1]$, then $2 \sin^{-1} x+3 \cos^{-1} x$ is equal to

KCET 2024

Options:

A. $\frac{4-6\pi}{5}$

B. $\frac{6\pi-4}{5}$

C. $\frac{3\pi}{2}$

D. 0

Answer: B

Solution:

$$\begin{aligned} & \because 2 \sin^{-1} x-3 \cos^{-1} x=4 \\ & \Rightarrow \quad 2 \sin^{-1} x-3\left(\frac{\pi}{2}-\sin^{-1} x\right)=4 \\ & \qquad \left[\because \cos^{-1} x=\frac{\pi}{2}-\sin^{-1} x\right] \end{aligned}$$

$$\Rightarrow \quad 5\sin^{-1}x = 4 + \frac{3\pi}{2}$$

$$\therefore \quad \sin^{-1}x = \frac{4+\frac{3\pi}{2}}{5} = \frac{8+3\pi}{10} \quad \ldots \text{(i)}$$

So, $2\sin^{-1}x + 3\cos^{-1}x$

$$= 2\sin^{-1}x + 3\left(\frac{\pi}{2} - \sin^{-1}x\right)$$

$$= -\sin^{-1}x + \frac{3\pi}{2}$$

$$= \frac{-8-3\pi}{10} + \frac{3\pi}{2} = \frac{-8-3\pi+15\pi}{10}$$

$$= \frac{12\pi-8}{10} = \frac{6\pi-4}{5}$$

Question6

If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ where $a, x \in (0,1)$, then the value of x is

KCET 2023

Options:

A. $\frac{a}{2}$

B. $\frac{2a}{1+a^2}$

C. $\frac{2a}{1-a^2}$

D. 0

Answer: C

Solution:

Here,

$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

We know that

$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

and $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\because 2\tan^{-1}a + 2\tan^{-1}a = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$2\left[\tan^{-1}\left(\frac{a+a}{1-a\times a}\right)\right] = 2\tan^{-1}x$

$\Rightarrow \quad \frac{2a}{1-a^2} = x$

Question7

The value of $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$**, where** $x \in \left(0, \frac{\pi}{4}\right)$ **is**

KCET 2023

Options:

A. $\frac{x}{2} - \pi$

B. $\pi - \frac{x}{3}$

C. $\pi - \frac{x}{2}$

D. $\frac{x}{2}$

Answer: C

Solution:

$$\text{Here, } \cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1-\sin x}}\right]$$

$$= \cot^{-1}\left[\frac{(\sqrt{1-\sin x}+\sqrt{1+\sin x})^2}{(1-\sin x)-(1+\sin x)}\right]$$

$$= \cot^{-1}\left(\frac{1-\sin x+1+\sin x+2\sqrt{1-\sin^2 x}}{1-\sin x-1-\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{2+2\sqrt{\cos^2 x}}{-2\sin x}\right)$$

$$\Rightarrow \cot^{1}\left(\frac{1+\cos x}{-\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{1+2\cos^2\frac{x}{2}-1}{-2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{1}\left(\frac{-\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(-\cot\frac{x}{2}\right) = \cot^{-1}\left(\cot\left(\pi-\frac{x}{2}\right)\right)$$

$$= \pi - \frac{x}{2}$$

Question8

Domain $\cos^{-1}[x]$ is, where [ ] denotes a greatest integer function

KCET 2022

Options:

A. $(-1, 2]$

B. $(-1, 2)$

C. $[-1, 2]$

D. $[-1, 2)$

Answer: D

Solution:

Let $y = \cos^{-1}[x]$

We know, for $\cos^{-1} x$

$\Rightarrow \quad -1 \leq x \leq 1$

Therefore, for $\cos^{-1}[x]$

$\Rightarrow \quad -1 \leq [x] \leq 1 \Rightarrow -1 \leq x < 2 \Rightarrow x \in [-1, 2)$

Question9

$\cos\left[\cot^{-1}(-\sqrt{3}) + \frac{\pi}{6}\right]$ is equal to

KCET 2021

Options:

A. 0

B. 1

C. $\frac{1}{\sqrt{2}}$

D. -1

Answer: D

Solution:

Given,

$$\begin{aligned}
&\cos\left[\cot^{-1}(-\sqrt{3}) + \frac{\pi}{6}\right] \\
&= \cos\left[\pi - \cot^{-1}(\sqrt{3}) + \frac{\pi}{6}\right] \\
&\quad [\because \cot^{-1} -x = \pi - \cot^{-1} x] \\
&= \cos\left[\pi - \frac{\pi}{6} + \frac{\pi}{6}\right] \\
&= \cos[\pi] = -1
\end{aligned}$$

Question10

$\tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\frac{5\pi}{2}\right]\sin^{-1}\left[\cos\left(\sin^{-}\frac{\sqrt{3}}{2}\right)\right]$ **is equal to**

KCET 2021

Options:

A. $\left(\frac{\pi}{6}\right)^2$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. π

Answer: A

Solution:

Given,

$$=\tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\frac{5\pi}{2}\right]\sin^{-1}\left[\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$$

$$=\tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\left(2\pi+\frac{\pi}{2}\right)\right]\sin^{-1}\left[\cos\left(\frac{\pi}{3}\right)\right]$$

$$=\tan^{-1}\left[\frac{1}{\sqrt{3}}\sin\frac{\pi}{2}\right]\cdot\sin^{-1}\left[\cos\frac{\pi}{3}\right]$$

$$=\tan^{-1}\left[\frac{1}{\sqrt{3}}\times 1\right]\cdot\sin^{-1}\left[\frac{1}{2}\right]$$

$$=\frac{\pi}{6}\times\frac{\pi}{6}=\left(\frac{\pi}{6}\right)^2$$

Question11

If $f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$**, then** $f'(\sqrt{3})$ **is**

KCET 2020

Options:

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{3}}$

D. $-\frac{1}{\sqrt{3}}$

Answer: B

Solution:

We have, $f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

On putting

$x=\tan\theta$
$\theta=\tan^{-1}x$

Then we get, $f(x)=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$=\sin^{-1}(\sin 2\theta)$
$=2\theta=2\tan^{-1}x$ .... (i)

On differentiating Eq. (i) both sides w.r.t. x, we get

$$f'(x)=\frac{2}{1+x^2}$$

$$\therefore \quad f'(\sqrt{3})=\frac{2}{1+(\sqrt{3})^2}=\frac{2}{1+3}=\frac{2}{4}=\frac{1}{2}$$

Question12

The domain of the function defined by $f(x)=\cos^{-1}\sqrt{x-1}$ is

KCET 2020

Options:

A. $[1,2]$

B. $[0,2]$

C. $[-1,1]$

D. $[0,1]$

Answer: A

Solution:

We have, $f(x)=\cos^{-1}\sqrt{x-1}$

$f(x)$ is defined when $\sqrt{x-1}\in[0,1]$

$$\begin{aligned} \therefore \quad & 0\le\sqrt{x-1}\le 1 \\ & 0\le x-1\le 1 \\ & 1\le x\le 2 \end{aligned}$$

$\therefore$ Domain of $f(x)$ is $[1,2]$.

Question13

The value of $\cos\left(\sin^{-1}\frac{\pi}{3}+\cos^{-1}\frac{\pi}{3}\right)$ isDoes not exist

KCET 2020

Options:

A. 0

B. 1

C. -0

D. Does not exist

Answer: D

Solution:

We have, $\cos\left(\sin^{-1}\frac{\pi}{3}+\cos^{-1}\frac{\pi}{3}\right)$

$\sin^{-1}x$ is defined $x\in[-1,1]$ but $\frac{\pi}{3}\notin[-1,1]$

$\therefore\cos\left(\sin^{-1}\left(\frac{\pi}{3}\right)+\cos^{-1}\left(\frac{\pi}{3}\right)\right)$ does not exists.

Question14

If $f(x)=\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ then $f'(0)=$

KCET 2019

Options:

A. $\frac{2\log 2}{5}$

B. $2\log 2$

C. $\frac{4\log 2}{5}$

D. $\log 2$

Answer: D

Solution:

We have, $f(x)=\sin^{-1}\left(\frac{2^{x+1}}{1+4x}\right)$

$\Rightarrow f(x)=\sin^{-1}\left(\frac{2^x\cdot 2}{1+(2^x)^2}\right)\Rightarrow f(x)=2\tan^{-1}(2^x)$

On differentiating both sides, we get

$$f'(x)=2\cdot\frac{1}{1+(2^x)^2}\cdot 2^x\log 2\left(\because\frac{d}{dx}(a^x)=a^x\log a\right)$$
$$\therefore f'(0)=\frac{2}{1+(1)^2}\cdot\log 2=\log 2$$

Question15

$\cos\left[2\sin^{-1}\frac{3}{4}+\cos^{-1}\frac{3}{4}\right]=$

KCET 2019

Options:

A. $\frac{3}{5}$

B. $\frac{-3}{4}$

C. $\frac{3}{4}$

D. does not exist

Answer: B

Solution:

$$\begin{aligned}
&\cos\left[2\sin^{-1}\frac{3}{4}+\cos^{-1}\frac{3}{4}\right]\\
&\Rightarrow\cos\left[\sin^{-1}\frac{3}{4}+\frac{\pi}{2}\right]\quad\left(\because\sin^{-1}x+\cos^{-1}y=\frac{\pi}{2}\right)\\
&\Rightarrow-\sin\left(\sin^{-1}\frac{3}{4}\right)\Rightarrow-\frac{3}{4}
\end{aligned}$$

Question16

If $a+\frac{\pi}{2}<2\tan^{-1}x+3\cot^{-1}x<b$ then 'a' and 'b' are respectively.

KCET 2019

Options:

A. 0 and 2π

B. 0 and π

C. $\frac{-\pi}{2}$ and $\frac{\pi}{2}$

D. $\frac{\pi}{2}$ and 2π

Answer: D

Solution:

$$\text{We have, } a+\frac{\pi}{2}<2\tan^{-1}x+3\cot^{-1}x<b$$
$$\Rightarrow \quad a+\frac{\pi}{2}<2\tan^{-1}x+3\left(\frac{\pi}{2}-\tan^{-1}x\right)<b$$
$$\Rightarrow \quad a+\frac{\pi}{2}<\frac{3\pi}{2}-\tan^{-1}x<b$$
$$\Rightarrow \quad a+\frac{\pi}{2}-\frac{3\pi}{2}<-\tan^{-1}x<b-\frac{3\pi}{2}$$
$$\Rightarrow \quad a-\pi<-\tan^{-1}x<b-\frac{3\pi}{2}$$
$$\Rightarrow \quad \pi-a>\tan^{-1}x>\frac{3\pi}{2}-b$$
$$\text{Since,} \quad -\frac{\pi}{2}<\tan^{-1}x<\frac{\pi}{2} \therefore a=\frac{\pi}{2}, b=2\pi$$

Question17

If $\sin^{-1}x+\cos^{-1}y=\frac{2\pi}{5}$, then $\cos^{-1}x+\sin^{-1}y$ is

KCET 2018

Options:

A. $\frac{2\pi}{5}$

B. $\frac{3\pi}{5}$

C. $\frac{4\pi}{5}$

D. $\frac{3\pi}{10}$

Answer: B

Solution:

To solve the given problem, we use the properties of inverse trigonometric functions. The problem states:

$\sin^{-1}x+\cos^{-1}y=\frac{2\pi}{5}$

We know a useful identity in trigonometry: $\sin^{-1} a + \cos^{-1} a = \frac{\pi}{2}$ for any sine or cosine value a.

Rewriting the given equation, we can express $\sin^{-1} x$ and $\cos^{-1} y$ in terms of $\frac{\pi}{2}$:

$\sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x$

$\cos^{-1} y = \frac{\pi}{2} - \sin^{-1} y$

Substitute these into the original equation:

$\left(\frac{\pi}{2} - \cos^{-1} x\right) + \left(\frac{\pi}{2} - \sin^{-1} y\right) = \frac{2\pi}{5}$

Simplifying:

$\pi - (\cos^{-1} x + \sin^{-1} y) = \frac{2\pi}{5}$

Solving for $\cos^{-1} x + \sin^{-1} y$:

$\cos^{-1} x + \sin^{-1} y = \pi - \frac{2\pi}{5}$

Which simplifies to:

$\cos^{-1} x + \sin^{-1} y = \frac{3\pi}{5}$

Thus, the value of $\cos^{-1} x + \sin^{-1} y$ is $\frac{3\pi}{5}$.

Question18

The value of the expression $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$ is

KCET 2018

Options:

A. $2 - \sqrt{5}$

B. $\sqrt{5} - 2$

C. $\frac{\sqrt{5}-2}{2}$

D. $5 - \sqrt{2}$

Answer: B

Solution:

To find the value of $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$, we can use the half-angle identity for tangent, which is:

$\tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x}$

Applying this identity, we get:

$\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right) = \frac{1-\cos\left(\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)}{\sin\left(\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)}$

Since $\cos(\cos^{-1}(z)) = z$, we have:

$\cos\left(\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right) = \frac{2}{\sqrt{5}}$

Now, to find the sine value, note that if $\cos\theta = \frac{2}{\sqrt{5}}$, then:

$\sin\theta = \sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2} = \sqrt{1-\frac{4}{5}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$

Substituting these values back into the formula for the half-angle tangent:

$\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right) = \frac{1-\frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{5}}}$

Simplifying the expression:

$= \frac{\sqrt{5}-2}{1} = \sqrt{5}-2$

Thus, the value is $\sqrt{5}-2$.

Question19

The range of $\sec^{-1}x$ is

KCET 2017

Options:

A. $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

B. $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

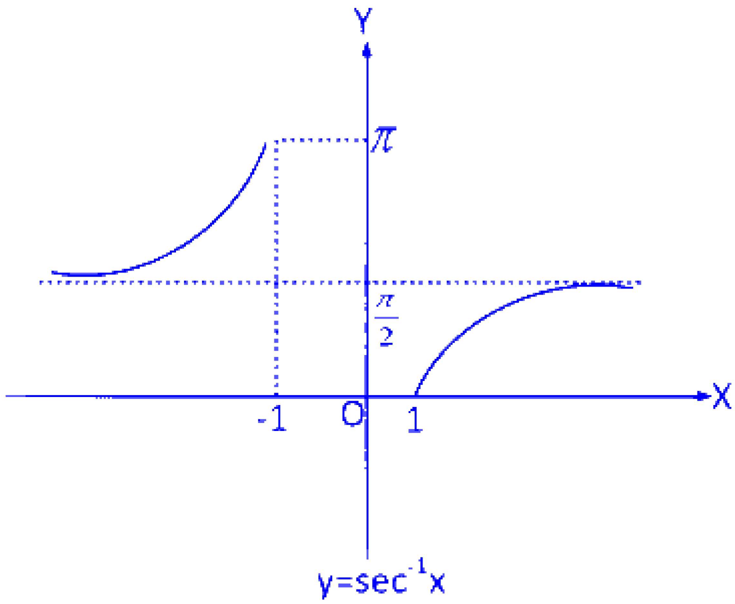
C. $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

D. $[0,\pi]$

Answer: A

Solution:

The range of $\sec^{-1} x$.
$= [0, \pi] - \left\{\frac{\pi}{2}\right\}$



$y = \sec^{-1} x$

Question20

If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, then $\cot^{-1} x + \cot^{-1} y$ is equal to

KCET 2017

Options:

A. $\frac{2\pi}{5}$

B. $\frac{\pi}{5}$

C. $\frac{3\pi}{5}$

D. π

Answer: B

Solution:

To solve the given problem, we'll use the following relationships:

$\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$

We know that $\tan^{-1}z + \cot^{-1}z = \frac{\pi}{2}$.

By expressing $\tan^{-1}x$ and $\tan^{-1}y$ in terms of their cotangent inverses, we have:

$\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$

$\tan^{-1}y = \frac{\pi}{2} - \cot^{-1}y$

Substituting these into the original equation:

$\left(\frac{\pi}{2} - \cot^{-1}x\right) + \left(\frac{\pi}{2} - \cot^{-1}y\right) = \frac{4\pi}{5}$

Simplifying the left-hand side:

$\pi - (\cot^{-1}x + \cot^{-1}y) = \frac{4\pi}{5}$

To find $\cot^{-1}x + \cot^{-1}y$, solve as follows:

$\cot^{-1}x + \cot^{-1}y = \pi - \frac{4\pi}{5}$

This simplifies to:

$\cot^{-1}x + \cot^{-1}y = \frac{\pi}{5}$

Therefore, the value of $\cot^{-1}x + \cot^{-1}y$ is $\frac{\pi}{5}$.
